Power Corrections and Rapidity Logarithms in Soft-Collinear Effective Theory

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Main Thesis Contents

Three papers form the main contents of the thesis

Goerke and **MIW**, *Renormalization of Dijet Operators at Order $1/Q^2$ in Soft-Collinear Effective Theory*, JHEP **04** (2018)

MIW, Luke, and Spourdalakis, *Rapidity Logarithms in SCET Without Modes*, Nucl. Phys. A **1014** (2021)

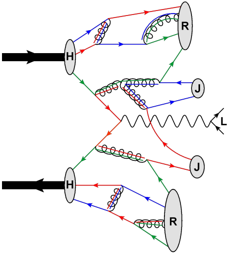
MIW, Luke, Roy, and Spourdalakis, *Factorization of Power Corrections in the Drell-Yan Process in EFT*, Phys. Rev. D **104** (2021)

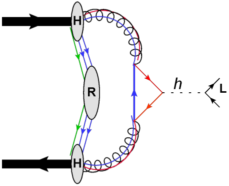
The Story

- Why Study Drell-Yan at Next-to-Leading Power
- Hard-Scale Running at LP and NLP
- Soft Matching at LP
- Soft Matching at NLP

The Drell-Yan Process

Back when QCD was new we wanted to study QCD itself using observables like the Drell-Yan process ($p\bar{p} \to V^* + X \to \ell\bar{\ell} + X$)

DY: 

$h \to \ell\bar{\ell}$: 

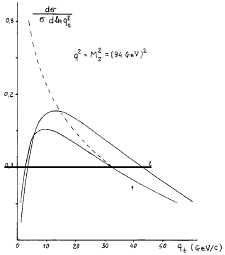
Nowadays we want to study the properties of the newly discovered Higgs. For $p\bar{p} \to h + X \to \ell\bar{\ell} + X$ the DY process is a source of background noise for the desired signal [ATLAS-CONF-2019-028]

The CSS approach to DY: successes

Original CSS approach to describing QCD processes was very successful.

$$d\sigma_{DY} \sim \sum_{ab} H_{ab}(Q) C_a(q_T, \zeta_A) C_b(q_T, \zeta_B) \otimes f_{a/N_1} f_{b/N_2} + O\left(\frac{q_T^2}{Q^2}\right)$$

H_{ab} and C_a can be calculated perturbatively when $q_T \gg \Lambda_{QCD}$. But also when $q_T \ll Q$, fixed-order calculations predict divergent behaviour.



[DDT, 1980]

Resummation fixes this, and can now achieve N^{3}LL at LP [Bizon 2018]

$$H_{ab}^{resum}(Q) \sim \exp\left(-\alpha_s \log^2 \frac{Q^2}{\mu_T^2}\right)$$

$$C_a^{resum}(q_T, \zeta_A) C_b^{resum}(q_T, \zeta_B) \sim \exp\left(-\alpha_s \log \frac{Q^2}{\mu_T^2} \log \frac{\mu_T^2}{q_T^2}\right)$$

The CSS approach to DY: drawbacks

$$d\sigma_{DY} \sim \sum_{ab} H_{ab}(Q) C_a(q_T, \zeta_A) C_b(q_T, \zeta_B) \otimes f_{a/N_1} f_{b/N_2} + O\left(\frac{q_T^2}{Q^2}\right)$$

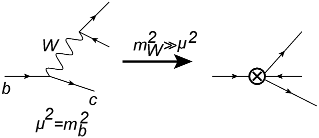
If we want more precision, we want to study the perturbative power corrections. However, it's very difficult to proceed further within the standard CSS formalism

Alternative approach: when large hierarchies of scales are present (like $q_T^2 \ll Q^2$), Effective Field Theories provide a powerful framework for deriving and exploiting factorization theorems

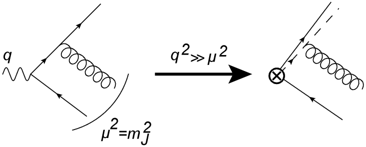
EFTs for jet-like physics

When colored particles are highly energetic and highly collimated, the appropriate EFT is Soft-Collinear Effective Theory

For comparison, the 4-Fermi EFT



... and SCET



Our formulation of SCET is constructed from multiple copies of QCD for each jet. Each copy models the other jet as lightlike color sources we call Wilson Lines

Factorizing the Drell-Yan process in SCET

Focusing on the photon-mediated Drell-Yan process, the reaction is mediated by the color-singlet vector current. In QCD,

$$J^\mu_{QCD}(x) = \bar{\psi}(x)\gamma^\mu\psi(x)$$

In SCET this current has an expansion in $1/Q$ as

$$J^\mu_{SCET}(x) = \sum_i \frac{1}{Q^{[i]}} C_{2i}(Q,\mu) O^\mu_{2(i)}(x,\mu)$$

The cross section is then

$$d\sigma_{DY} \propto \int d^d x \, e^{-iQ\cdot x} L_{\mu\nu} \langle N_1 N_2 | J^{\mu\dagger}_{QCD}(x) J^\nu_{QCD}(0) | N_1 N_2 \rangle$$

$$d\sigma_{DY} \propto \int d^d x \, e^{-iQ\cdot x} \sum_{ij} \frac{C^\dagger_{2i} C_{2j}}{Q^{[i]+[j]}} \langle N_1 N_2 | O^{\dagger\mu}_{2(i)}(x) O_{2(j)\mu}(0) | N_1 N_2 \rangle$$

Matrix elements of effective operators only generate infrared energy scales*, so

$$d\sigma_{DY} \propto \sum_{ij} \frac{H_{ij}(Q,\mu)}{Q^{[i]+[j]}} S_{ij}(\mu, q_T)$$

Note: Different notions of factorization!

Usual SCET: $O_{\text{QCD}} \to C_H O_{\text{SCET}}$

$$\mathcal{L}_{\text{SCET}} = C_H(Q) O_{\text{SCET}} + \mathcal{L}_n + \mathcal{L}_{\bar{n}} + \mathcal{L}_s$$

so e.g. matrix elements take the form

$$\begin{aligned}\langle X| O_{\text{QCD}} |0\rangle &= C_H(Q) \langle X_n| O_n |0\rangle \\ &\times \langle X_{\bar{n}}| O_{\bar{n}} |0\rangle \langle X_s| O_s |0\rangle \\ &= C_H(Q) J_n(\lambda Q) \\ &\times J_{\bar{n}}(\lambda Q) S(\lambda^2 Q)\end{aligned}$$

From EFT defined at Q, immediately get factorization of all infrared scales simply from decoupling of modes

Our Formulation

$$\mathcal{L}_{\text{SCET}} = C_H(Q) O_{\text{SCET}} + \mathcal{L}_n + \mathcal{L}_{\bar{n}}$$

$$\langle X| O_{\text{QCD}} |0\rangle = C_H(Q) \langle X_n X_{\bar{n}}| O_{\text{SCET}} |0\rangle$$

Then $O_{\text{SCET}} \to C_J O_{\text{soft}}$

$$\langle X| O_{\text{QCD}} |0\rangle = C_H(Q) C_J(\mu_J) \langle X_s| O_{\text{soft}} |0\rangle$$

Then $O_{\text{soft}} \to C_S \mathbb{1}$

$$\langle X| O_{\text{QCD}} |0\rangle = C_H(Q) C_J(\mu_J) C_S(\mu_S)$$

Get factorization from matching steps at each relevant energy scale. Everything is matching!

Problems with scale setting

Would like to set $\mu \sim q_T$ in

$$d\sigma_{DY} \propto \sum_{ij} \frac{H_{ij}(Q,\mu)}{Q^{[i]+[j]}} S_{ij}(\mu, q_T)$$

but we calculate that, e.g. at LP,

$$H_{0,0}(Q,\mu = q_T) = 1 + \frac{\alpha_s C_F}{2\pi}\left(-\log^2 \frac{Q^2}{q_T^2} + 3\log\frac{Q^2}{q_T^2}\right) + \dots$$

so there are large logarithms of Q^2/q_T^2 if $\mu \sim q_T$, and need to resum the logs to regain perturbative control. (New effective coupling becomes $\alpha_s \log Q^2/q_T^2 \sim 1$)

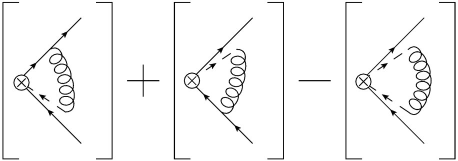
$$H_{ij}(Q,\mu = q_T) \sim \exp\left(\int_Q^{q_T} d\log\mu \ \gamma_{H_{ij}}\right) H_{ij}(Q, \mu = Q) \ .$$

Paper 1 – Renormalization of the $O_{2(i)}$

At leading power, the only operator is

$$O^{\mu}_{2(0)}(x) = \bar{\chi}_n(x_n)\gamma^{\mu}\chi_{\bar{n}}(x_{\bar{n}})$$

One-loop diagrams are



Find

$$Z_{2(0)} = 1 + \frac{\alpha_s C_F}{4\pi}\left[\frac{2}{\epsilon^2} + \frac{1}{\epsilon}\left(3 - 2\log\frac{Q^2}{\mu^2}\right)\right] + \dots$$

and

$$\gamma_{C_{2,0}} = \frac{\alpha_s C_F}{2\pi}\left(2\log\frac{Q^2}{\mu^2} - 3\right)$$

[Manohar 2002]

The NLP Scattering Operators

In [Freedman 2014] and Paper 1 we found all subleading operators up to $1/Q^2$ which are relevant for 2-sector observables (with up to one real gluon emission)

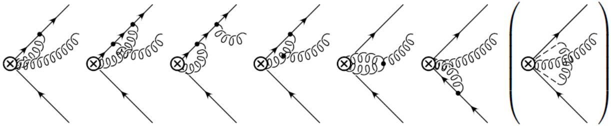
$$O^{\mu}_{2(1A_1)} = \bar{\chi}_n(x_n)\mathcal{B}^{\rho}_n(x+\bar{n}t)\gamma^{\mu}\frac{\not{\bar{n}}}{2}\gamma^{\perp}_{\rho}\chi_{\bar{n}}(x_{\bar{n}})$$

$$\cdots$$

$$O^{\mu}_{2(2A_1)} = \bar{\chi}_n(x_n)\mathcal{B}^{\rho\sigma}_n(x+\bar{n}t)\gamma^{\mu}\gamma^{\perp}_{\rho}\gamma^{\perp}_{\sigma}\chi_{\bar{n}}(x_{\bar{n}})$$

$$O^{\mu}_{2(2A_2)} = \bar{\chi}_n(x_n+\bar{n}t)\mathcal{B}^{\rho\sigma}_n(x)\gamma^{\perp}_{\rho}\frac{\eta}{2}\gamma^{\mu}\frac{\not{\bar{n}}}{2}\gamma^{\perp}_{\sigma}\frac{\not{\bar{n}}}{2}\gamma^{\perp}_{\rho}\chi_{\bar{n}}(x_{\bar{n}})$$

These have 1-loop diagrams like



Renormalization Results

After calculating all the 1-loop diagrams for each operator, the new anomalous dimensions found in Paper 1 are

$$
\begin{aligned}
\gamma_2^{(2A_1)}(u,v) &= \frac{\alpha_s}{\pi}\delta(u-v)\left[C_F\left(\log\frac{-Q^2}{\mu^2}+\log(\bar{v})-\frac{3}{2}\right)+\frac{C_A}{2}\left(\log\frac{v}{\bar{v}}+\frac{5}{2}\right)\right] \\
&\quad+\frac{\alpha_s}{\pi}\left(C_F-\frac{C_A}{2}\right)\frac{1}{v\bar{v}^2}\left(\bar{u}^2\bar{v}^2\,\theta(u+v-1)+uv(\bar{u}\bar{v}+\bar{u}+\bar{v}-1)\theta(1-u-v)\right) \\
&\quad-\frac{\alpha_s}{\pi}\frac{C_A}{2}\frac{1}{v\bar{v}^2}\Bigg(v\bar{u}^2(1+\bar{v})\theta(u-v)+u\bar{v}^2(1+\bar{u})\theta(v-u) \\
&\qquad+\left[v\bar{u}^2\frac{\theta(u-v)}{u-v}+u\bar{v}^2\frac{\theta(v-u)}{v-u}\right]_+\Bigg) \\
\gamma_2^{(2A_2)}(u,v) &= \frac{\alpha_s}{\pi}\delta(u-v)\left[C_F\left(\log\frac{-Q^2}{\mu^2}+\log(v)-\frac{3}{2}\right)+\frac{C_A}{2}\left(\log\frac{\bar{v}}{v}+\frac{5}{2}\right)\right] \\
&\quad+\ldots
\end{aligned}
\tag{1}
$$

So now we can, in principle, run all the operators down to $O_2(\mu=q_T)$. These RGE equations, with complicated mixing between continuous labels u and v, are very difficult to solve. Can at least use LL approximation to get preliminary results

Evaluating the Soft Factor S_{ij}

Recall

$$d\sigma_{DY} \propto \sum_{ij} \frac{H_{ij}(Q,\mu)}{Q^{[i]+[j]}} S_{ij}(\mu, q_T)$$

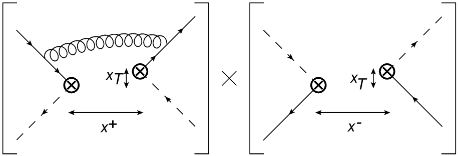
Now that we can run $H_{ij}(Q,\mu)$ down to $\mu = q_T$, how do we calculate S_{ij}? After all,

$$S_{ij} = \int d^d x\, e^{-iQ\cdot x} \langle N_1 N_2 |\, O^{\mu}_{2(i)}(x) O_{2(j)\mu}(0)\, | N_1 N_2 \rangle$$

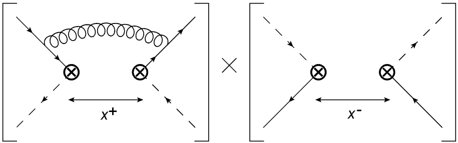
is a matrix element between non-perturbative states, need to find a matching to proceed further.

The Soft Matching

In the parton model, the product $O^{\dagger\mu}_{2(i)}O_{2j\mu}$ has 1-loop graphs like



When working at $\mu = q_T$, q_T is now considered to be large (so x_T is small), and thus can do OPE in x_T



We get the usual PDFs (as seen in DIS)! We write

$$\int d^dx\ O^{\dagger\mu}_{2(i)}(x)O_{2(j)\mu}(0) \to C_{S,(ij)} \otimes O_q O_{\bar{q}}, \quad \langle N_1|\, O_q\, |N_1\rangle = f_{q/N_1}$$

C_S at LP in the literature: the *

At LP, one finds [Becher and Neubert 2010]

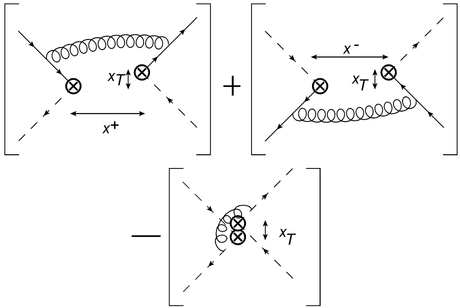
$$C_{S,(0,0)} \propto \delta(q_T) + \frac{\alpha_s C_F}{2\pi}\left(\log\frac{Q^2}{\mu^2}\left[\frac{1}{q_T^2}\right]_+^{\mu^2} + \ldots\right)$$

Strange: the hard scale Q^2 seems to appear dynamically from matrix elements calculated in the EFT

Literature explanation: Collinear modes contain information about large lightcone components q^-, q^+. Normally not allowed to appear in matrix elements because of boost invariance. When rapidity divergences $\int dk^-/k^-$ enter collinear diagrams, regulating these rapidity divergences breaks boost invariance, and these large lightcone components are then allowed to appear in the matrix element

Paper 2: Our Calculation of C_S

Our version of SCET is meant to be boost-invariant in *each* sector.
Our calculation of C_S, with no rapidity regulator, gives



Rescaling the loop momenta of the $\bar{n}$ graph gives

$$C_{S,(0,0)} \propto \delta(q_T) + \frac{\alpha_s C_F}{2\pi}\left(\log\frac{\nu^2}{\mu^2}\left[\frac{1}{q_T^2}\right]_+^{\mu^2} + \dots\right)$$

The ν doesn't cancel between graphs in our formalism – it replaces the previous dependence on Q^2

Our Explanation for the Q^2

Interpretation:

- SCET has no dynamical dependence on Q^2
- $\log(Q^2/\mu^2)$ is from QCD, so is our SCET wrong? No, it's just that $\nu = Q$ is a condition imposed by matching

More trustworthy calculation if repeated with a rapidity regulator. With a boost-invariant rapidity regulator:

$$C_{S,(0,0)} \propto \delta(q_T) + \frac{\alpha_s C_F}{2\pi}\left(\frac{1}{\eta_n} + \frac{1}{\eta_{\bar{n}}} + \log\frac{\nu^2}{\mu^2}\left[\frac{1}{q_T^2}\right]_+^{\mu^2} + \dots\right)$$

We find a rapidity counterterm, from which we can sum rapidity logarithms! $C_S(\mu, \nu = q_T) = \exp(\int_Q^{q_T} d\log\nu\,\gamma_\nu) \otimes C_S(\mu, \nu = Q)$

Paper 3: C_S at NLP

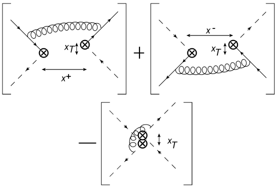
Recall:

$$\int d^d x \; O^{\dagger\mu}_{2(i)}(x) O_{2(j)\mu}(0) \to C_{S,(ij)} \otimes O_q O_{\bar{q}}, \quad \langle N_1 | \, O_q \, | N_1 \rangle = f_{q/N_1}$$

so the DY cross section is

$$d\sigma_{DY} \propto \sum_{ij} \frac{H_{ij}(Q, \mu = q_T)}{Q^{[i]+[j]}} C_{S,ij}(\mu = q_T, \nu = q_T) \otimes f_{q/N_1} f_{q/N_2}$$

Same types of 1-loop diagrams

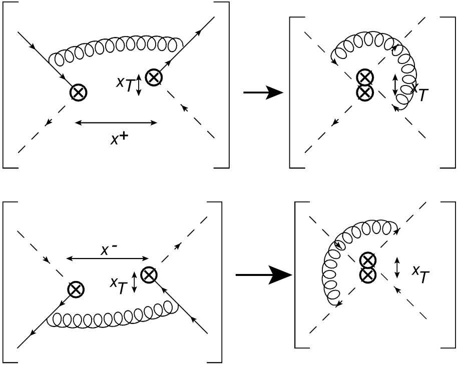


Straightforward to calculate but now there are new problems to solve

- Overcounting probability is more difficult to correct
- Subleading operators mix in rapidity with leading-power operators

Overlap Subtraction is Harder

At LP there is one operator-product combination, $O^{\dagger}_{2(0)} O_{2(0)}$, with soft limits



These *n* and *nb* soft limits give the same probability, so there is overcounting which must be subtracted away

At NLP, however, the same diagrams have different derivative insertions, so the soft limit are not the same

Overlap Subtraction Prescription

What we found:

- Subtract off half the soft limit of each diagram $\mathcal{M}_i = \tilde{\mathcal{M}}_i - \frac{1}{2}\mathcal{M}_{i,soft}$
- Need subleading contributions from LP soft limits!
- Can show then that the net result of all operators is rapidity-finite (like QCD!)
- We also find that the same rapidity ambiguity is present at NLP, $\log \nu^2/\mu^2$ when matched onto QCD is $\log Q^2/\mu^2$

Rapidity Renormalization at NLP

Need mixing with leading-power operator

We find, e.g.

$$Z_{(1A_1,1A_2),(0,0)} = -\frac{\alpha_s C_F}{2\pi^2}\frac{1}{\eta_n}\delta(\bar{z}_1)\delta(\bar{z}_2)\delta(u_1)\delta(u_2)$$

$$\cdots$$

$$Z_{(2A_1,0),(0,0)} = -\frac{\alpha_s C_F}{2\pi^2}\frac{1}{\eta_n}z_1 z_2 \delta'(\bar{z}_1)\delta(\bar{z}_2)\delta\left(u+\frac{\bar{z}_1}{z_1}\right)$$

$$\cdots$$

Since NLP products $O^{\dagger\mu}_{2(i)}O_{2(j)\mu}$ start at $O(\alpha_s)$, need to go to $O(\alpha_s^2)$ to find diagonal terms. All we know is how they mix with the LP operator. Still need diagonal terms for a full NLP rapidity summation

Summary of Work

Factorized cross section with quark initial states:

$$d\sigma_{DY} \propto \sum_{ij} \frac{H_{ij}(Q,\mu=Q)U_{ij}(Q,q_T)}{Q^{[i]+[j]}} V_{(ij),(k\ell)}(q_T,Q)$$
$$\otimes C_{S,(k\ell)}(\mu=q_T,\nu=\mu)\otimes f_{q/N_1} f_{\bar{q}/N_2}$$

$$U_{ij}(Q,q_T) \sim \exp\left(\int d\log\mu\ \gamma_{H_{ij}}\right)$$

$\gamma_{H_{ij}}$ for required $1/Q^2$ operators now known to 1-loop from Paper 1

$$V_{(ij),(k\ell)}(q_T,Q) \sim \exp\left(\int d\log\nu\ \gamma_{\nu(ij),(k\ell)}\ \otimes\right)$$

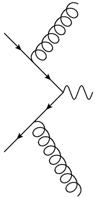
$\gamma_{\nu(ij),(k\ell)}$ known at LP to 1-loop from Paper 2, agrees with literature results

$\gamma_{\nu(ij),(k\ell)}$ at NLP, off-diagonals terms are known at 1-loop from Paper 3

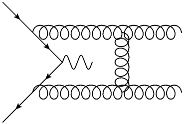
Thanks for listening!

Backup Slides: Possible Issues

DY at 2 loops is complicated. We have a supposedly all-orders result but... how do you account for the back-to-back jet configuration where the energetic jets have large p_T which mutually cancel to a small q_T?



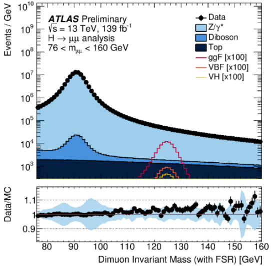
Glaubers are always a stumbling block for factorization proofs. QCD diagrams like



Can we reproduce the dynamics of these diagrams in SCET? They are known to cancel at LP, but no proofs of such a cancellation at NLP

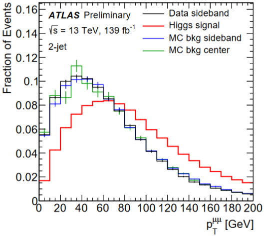
Backup Slides: Higgs to Leptons Spectrum + Background

Lepton-Pair Invariant Mass Distribution
- Note the large DY (Z/γ^*) background



[Zirui Wang, International Conference on Kaon Physics 2019]

Lepton-Pair p_T Spectrum
- Peak region needs resummation
- Right shoulder needs resummation and power corrections



[ATLAS-CONF-2019-028]

Backup Slides: Linear Power Corrections?

From [Ebert, Michel, Stewart, Tackmann, 2006.11382]

The natural way to satisfy both requirements is to expand the leptonic measurements neither in q_T nor p_L and thus keep the exact leptonic tensor,

$$\boxed{\frac{\mathrm{d}\sigma^{(0+L)}(\Theta)}{\mathrm{d}^4q\,\mathrm{d}\mathcal{O}} \equiv \frac{1}{2E_{\mathrm{cm}}^2} \sum_{i=-1,2,4,5} L_i(q,\mathcal{O},\Theta)\, W_i^{(0)}(q^2, s_{aq}, s_{bq})\,.} \tag{2.102}$$

In the following, we always use the notation $\mathrm{d}\sigma^{(0+L)}$ to denote the inclusion of the exact leptonic tensor as in eq. (2.102). By treating the leptonic tensor exactly, it in fact incorporates *all* fiducial power corrections that multiply the leading-power hadronic structure functions. The leptonic tensor does not produce small-q_T logarithms, which solely arise from the hadronic tensor. Therefore, eq. (2.102) automatically resums all logarithms in fiducial power corrections to the same order as the resummation is included for the hadronic tensor.

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